FULL-WAVE ANALYSIS OF SUPERCONDUCTING MICROSTRIP LINES ON SAPPHIRE SUBSTRATES

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ABSTRACT

A computationally efficient full-wave technique is developed to analyze superconducting microstrip lines on M-plane sapphire in which the optic axis is in the plane of the substrate at an arbitrary angle with respect to the propagation direction. To increase the efficiency of the method, the superconducting strip is replaced by an equivalent surface impedance which accounts for the loss and kinetic inductance of the superconductor. Complex propagation constant and characteristic impedance are calculated and compared to both measured results and results obtained by the more rigorous volume-integral-equation method.

I. INTRODUCTION

The uniform and low-loss dielectric properties of sapphire make it attractive for high-temperature superconducting passive microwave device applications. The performance of microwave devices on sapphire has been demonstrated in the form of a low-loss 9-ns delay line [1] and a 4-pole microstrip filter operating at 10 GHz [2]. However, the anisotropic dielectric property of sapphire requires special attention in the design of these devices.

Several authors have proposed full-wave analyses of superconducting planar transmission lines [3,4], but analysis of superconducting lines on anisotropic substrates is not available. In this paper, an efficient full-wave method is developed to calculate the effective dielectric constant, attenuation, and characteristic impedance of superconducting microstrip lines on uniaxial anisotropic substrates in which the optic axis is in the plane of substrate at an arbitrary angle with respect to the propagation direction. To increase the efficiency of the method, the superconducting strip is transformed into an infinitely thin strip using an equivalent surface impedance concept. This equivalent surface impedance accounts for the loss and kinetic inductance of the superconductor. A 2-D dyadic Green's function is used to formulate an integral equation for the surface current in the strip. Galerkin's method with entire-domain basis functions is used to solve for the complex propagation constant. The characteristic impedance is then determined by calculating the propagating power. Numerical results are compared to previously published data for the isotropic substrate material $LaAlO_3$.

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By utilizing the concept of equivalent surface impedance, the computation time, compared to the more rigorous volume-integral-equation method [4,5], is reduced by nearly two orders of magnitude.

II. METHOD OF SOLUTION

In Fig. 1, a superconducting strip is shown on an anisotropic substrate for which the optic axis is in the x-y plane at an angle θ with respect to the y-axis. The thickness of the substrate is assumed to be h. The thickness and the width of the strip are t and $2w$ respectively. The structure is uniform along the propagation direction. A perfectly conducting ground plane is assumed. The permittivity tensor $\bar{\bar{\epsilon}}$ for the substrate in the chosen coordinate system can be written as

$$\bar{\bar{\epsilon}} = \begin{bmatrix} \epsilon_\perp \cos^2\theta + \epsilon_\| \sin^2\theta & (\epsilon_\| - \epsilon_\perp)\sin\theta\cos\theta & 0 \\ (\epsilon_\| - \epsilon_\perp)\sin\theta\cos\theta & \epsilon_\perp \sin^2\theta + \epsilon_\| \cos^2\theta & 0 \\ 0 & 0 & \epsilon_\perp \end{bmatrix} \quad (1)$$

where $\epsilon_\perp$ and $\epsilon_\|$ are the permittivities perpendicular and parallel to the optic axis, respectively.

To implement an efficient method, an equivalent surface impedance Z_s is used to transform the superconducting strip with a finite thickness to an infinitely thin strip. Since the fields and current in the superconducting strip behave differently from those in the infinitely thin strip, the surface impedance derived for an infinite half-plane superconductor or for parallel plates is not suitable for the transformation. Instead, Z_s should preserve the power dissipated and stored

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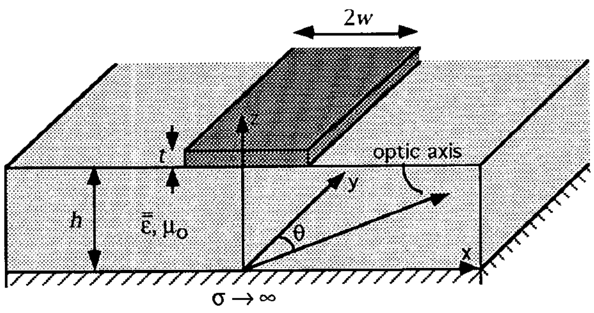

Figure 1: Superconducting microstrip line with a perfectly conducting ground plane on an M-plane sapphire substrate.

in the superconductor,

$$\frac{\int_h^{h+t}\int_{-w}^{w}|J_y^{(0)}|^2\,dx\,dz}{\sigma_{sc}\left|\int_h^{h+t}\int_{-w}^{w}J_y^{(0)}\,dx\,dz\right|^2}=\frac{\int_{-w}^{w}Z_s\,|J_{s,y}|^2\,dx}{\left|\int_{-w}^{w}J_{s,y}\,dx\right|^2} \tag{2}$$

where $J_y^{(0)}$ is the y-component of the volume current in the superconducting strip, σ_{sc} is the complex conductivity of the superconductor, and $J_{s,y}$ is the y-component of the surface current $\overline{J}_s$ in the infinitely thin strip. Throughout the analysis, the $\exp(-i\omega t)$ time dependence is used but suppressed.

The volume current $J_y^{(0)}$ remains nearly frequency independent as long as the penetration depth λ is much less than the skin depth δ. However, the exact distribution of current cannot be put in a simple analytic form. An approximate solution for $t \approx \lambda$ published in [6] is adopted for the derivation of Z_s.

The first-order approximation for the surface current $J_{s,y}$ can be taken as

$$J_{s,y} \propto \frac{1}{\sqrt{1-(x/w)^2}}. \tag{3}$$

Because the y-component of the electric field ($E_y = Z_s J_{s,y}$) is finite at the edges of the strip [7], Z_s should cancel the singularity of $J_{s,y}$ at the edges. Since $E_y \ll E_x$ and E_z, we can assume an uniform distribution of E_y across the strip without introducing significant error. Based upon the assumptions, the equivalent surface impedance is determined to be

$$Z_s(x) = \mathcal{Z}\sqrt{1-(x/w)^2} \tag{4}$$

where

$$\mathcal{Z} = \frac{A_0^2 t\pi}{4\sigma_{sc}}\left[\ln\left(\frac{w+x_o}{w-x_o}\right)+\frac{\lambda^2\left(e^{2(w-x_o)t/\lambda^2}-1\right)}{wt\left(1-(x_o/w)^2\right)}\right] \tag{5}$$

$$A_0 = \left[t\sin^{-1}\left(\frac{x_o}{w}\right)-\frac{\lambda^2\left(1-e^{(w-x_o)t/\lambda^2}\right)}{w\sqrt{1-(x_o/w)^2}}\right]^{-1} \tag{6}$$

$$x_o = w\sqrt{1+(\lambda^2/wt)^2}-\frac{\lambda^2}{2t}. \tag{7}$$

Once the superconducting strip is transformed into an infinitely thin strip with the surface impedance Z_s, a 2-D dyadic Green's function $\overline{\overline{\mathcal{G}}}(k_x, k_y)$ for layered, anisotropic media [8] is used to formulate an integral equation for the surface current in the strip,

$$Z_s(x)\overline{J}_s(x) = \frac{i\omega\mu_o}{2\pi}\int_{-\infty}^{\infty} dk_x\,\overline{\overline{\mathcal{G}}}(k_x, k_{y0})\cdot\overline{\mathcal{J}}_s(k_x)e^{ik_x x} \tag{8}$$

where k_{y0} is the complex propagation constant and $\overline{\mathcal{J}}_s(k_x)$ is the Fourier transform of $\overline{J}_s(x)$. The surface current is expressed in terms of a set of orthogonal basis functions,

$$J_{s,x}(x) = \sum_{n=0}^{N} a_n \sin[n\pi(x+w)/2w] \tag{9}$$

$$J_{s,y}(x) = \sum_{n=0}^{N} b_n \frac{\cos[n\pi(x+w)/2w]}{\sqrt{1-(x/w)^2}} \tag{10}$$

for $|x| \le w$.

A set of linear equations for the coefficients of the basis functions is obtained by applying Galerkin's method. The complex propagation constant k_{y0} is determined so that the matrix equation has a nontrival solution. The surface current is then obtained by searching for the eigenvector of the coefficient matrix corresponding to the "zero" eigenvalue. To determine the propagating power, the electric field $\overline{\mathcal{E}}(k_x, z)$ and magnetic field $\overline{\mathcal{H}}(k_x, z)$ in the spectral domain are obtained in terms of the surface current. The characteristic impedance Z_c of the line is then calculated using the power-current definition,

$$Z_c = \frac{1}{2\pi}\frac{\int_{-\infty}^{\infty}\int_0^{\infty}\hat{y}\cdot(\overline{\mathcal{E}}\times\overline{\mathcal{H}}^*)\,dz\,dk_x}{\left|\int_{-w}^{w}J_{s,y}\,dx\right|^2}. \tag{11}$$

III. NUMERICAL RESULTS

In the calculations, a low-order (N) series of basis functions is found to be sufficient for the frequency range studied in this work ($N = 0$ is sufficient for isotropic substrates, and $N = 2$ is required for anisotropic substrates).

Figure 2 shows good agreement between the calculated effective dielectric constants using the equivalent surface impedance approach discussed here, and the measured data [5] for three Nb microstrip lines on the isotropic substrate $LaAlO_3$ ($\epsilon = 23.5$). The parameters used for the calculation are those used in [5]. In Fig. 3, the quality factor Q and the characteristic impedance Z_c of a YBCO microstrip line, calculated using the equivalent surface impedance approach, are shown together with the numerical results obtained using the spectral-domain volume-integral-equation method (SDVIE) [4,5]. The comparison again shows good agreement, thereby validating the equivalent surface impedance approach. This approach achieves a great saving in the computation time,

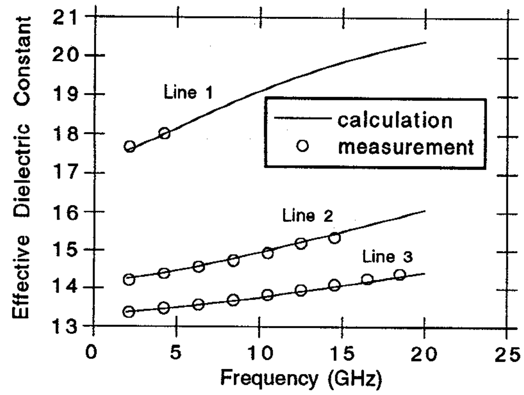

Figure 2: Measured and calculated effective dielectric constant of three Nb microstrip lines on $LaAlO_3$. Calculations were performed using the equivalent surface impedance approach ($\lambda = 0.0714$ μm; $\sigma_1 = 128$ S/μm; $h = 508$ μm; $t = 0.4$ μm; Line 1, $2w = 1800$ μm; Line 2, $2w = 181$ μm; and Line 3, $2w = 20$ μm).

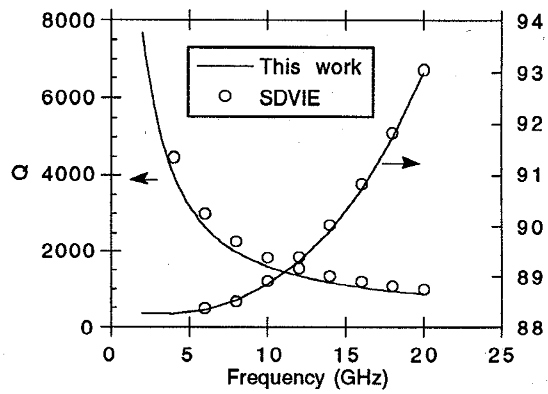

Figure 3: Calculated quality factor and characteristic impedance of a YBCO microstrip line on $LaAlO_3$. Comparison is made between the equivalent surface impedance approach and the spectral-domain volume-integral-equation method ($\lambda = 0.323$ μm, $\sigma_1 = 3.5$ S/μm, $h = 508$ μm, $t = 0.4$ μm, and $2w = 20$ μm).

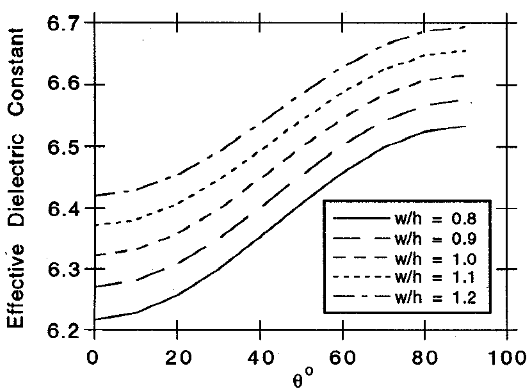

Figure 4: Calculated effective dielectric constant of YBCO microstrip lines on M-plane sapphire substrate at 2 GHz ($\lambda = 0.323$ μm, $\sigma_1 = 3.5$ S/μm, $h = 430$ μm, and $t = 0.4$ μm).

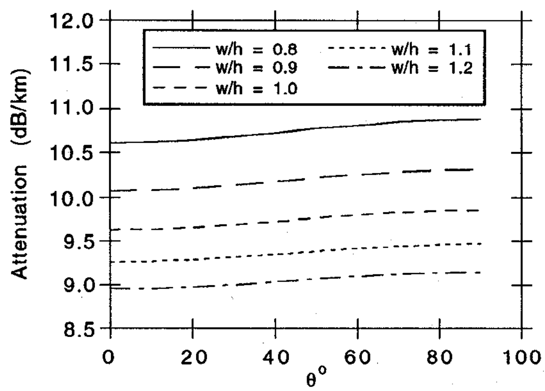

Figure 5: Calculated attenuation constant of YBCO microstrip lines on M-plane sapphire substrate at 2 GHz ($\lambda = 0.323$ μm, $\sigma_1 = 3.5$ S/μm, $h = 430$ μm, and $t = 0.4$ μm).

requiring nearly two orders of magnitude less time than the SDVIE method.

This efficient full-wave method using the equivalent surface impedance approach is then applied to study YBCO microstrip lines ($t = 0.4$ μm) on 430-μm-thick M-plane sapphire substrates ($\epsilon_\perp = 9.34$ and $\epsilon_\| = 11.6$). Typical values of $\lambda = 0.323$ μm and $\sigma_1 = 3.5$ S/μm for YBCO at 77 K are used. The calculated effective dielectric constant ϵ_{eff}, attenuation constant α, and characteristic impedance Z_c at 2 GHz for various w/h ratios are illustrated in Figs. 4-6. Because of the larger dielectric constant along the optic axis, the inductance L and capacitance C of the lines increase as θ increases from 0^o to 90^o. The effective dielectric constant, which is a function of the product of L and C, shows a greater dependence on θ compared to the attenuation constant and the characteristic impedance, which depend on the ratio of L and C. The dependence of ϵ_{eff}, α, and Z_c on θ is larger for smaller w/h because of the larger fringing fields.

IV. CONCLUSIONS

A computationally efficient full-wave method utilizing the concept of equivalent surface impedance is developed for characterization of superconducting microstrip line on anisotropic substrates. The case of M-plane sapphire is considered in which the optic axis of the substrate is in the plane of the substrate at an arbitrary angle with respect to the propaga-

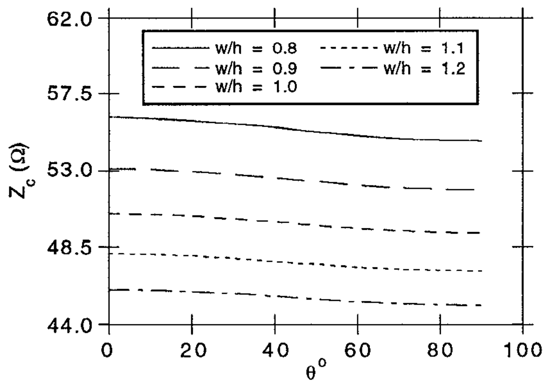

Figure 6: Calculated characteristic impedance of YBCO microstrip lines on M-plane sapphire substrate at 2 GHz ($\lambda = 0.323$ μm, $\sigma_1 = 3.5$ S/μm, $h = 430$ μm, and $t = 0.4$ μm).

tion direction. The calculated results show good agreement with previously published data and with the more rigorous volume-integral-equation method.

V. ACKNOWLEDGMENTS

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